

Fig. 1

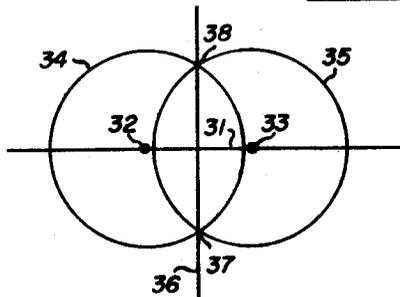


Fig. 2

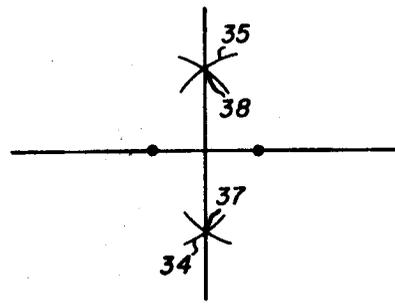


Fig. 2A

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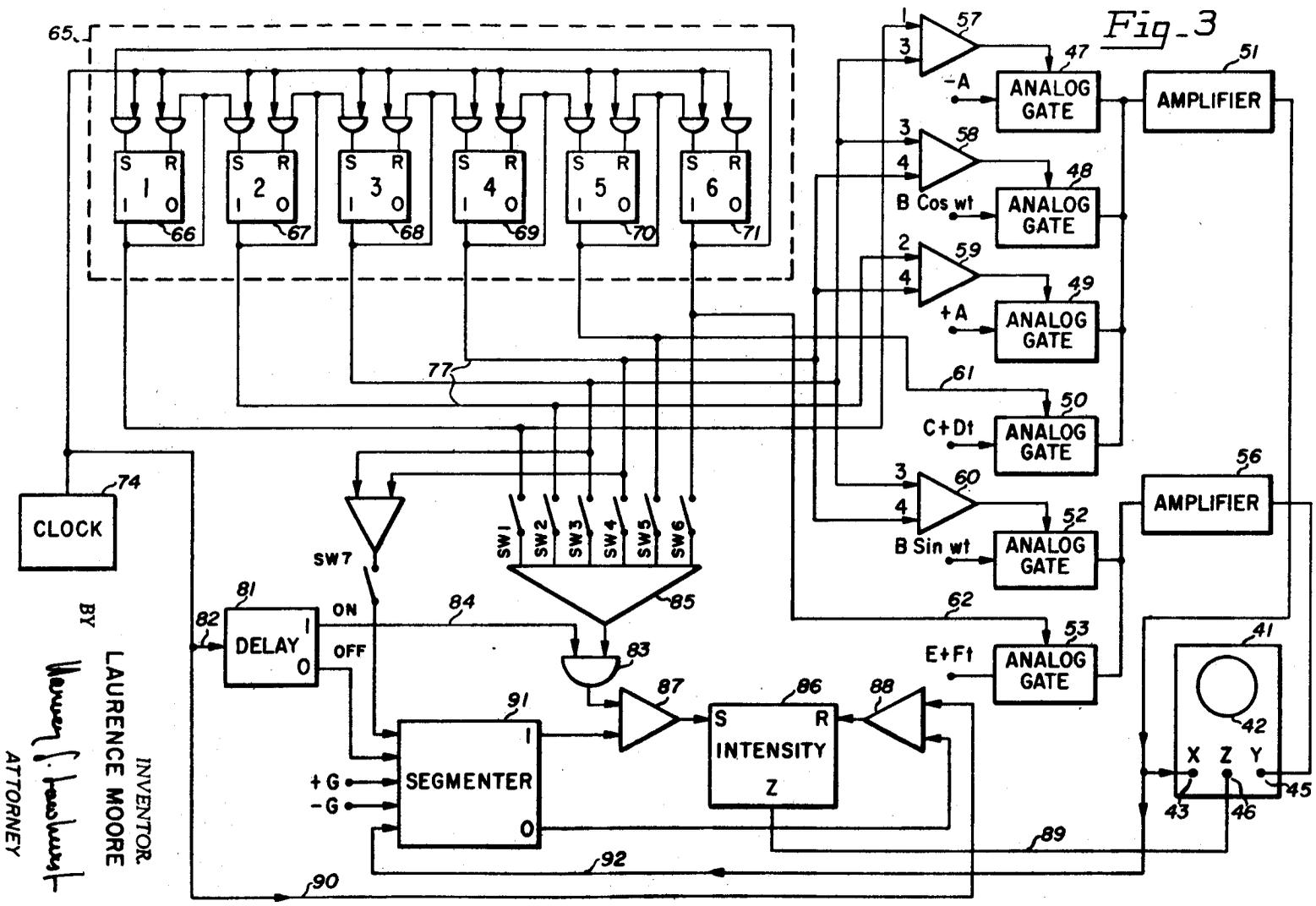
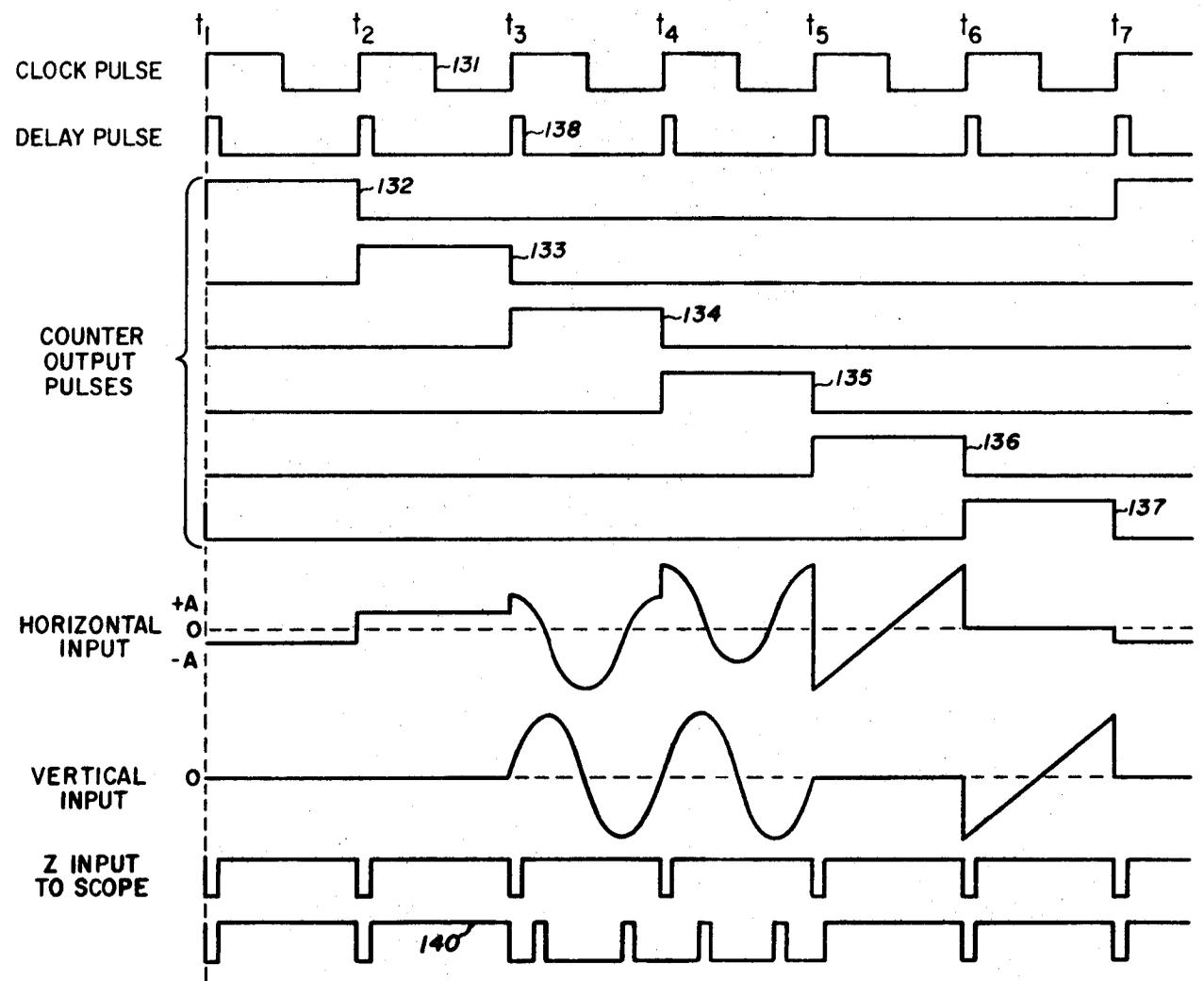


Fig-3

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 ELECTRIC APPARATUS FOR ANIMATING GEOMETRIC FIGURES AND  
 RELATIONSHIPS UTILIZING A CATHODE RAY TUBE DISPLAY  
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*Fig. 4*

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**ELECTRICAL APPARATUS FOR ANIMATING GEOMETRIC FIGURES AND RELATIONSHIPS UTILIZING A CATHODE RAY TUBE DISPLAY**

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This invention relates to electrical apparatus for providing visual animated geometric figures and relationships. More specifically, this invention relates to electrical apparatus for producing dynamic, animated algebraic, trigonometric or geometric relationships and figures on visual display means, such as a cathode ray tube oscilloscope, which dynamic figures and relationships can be permanently recorded for subsequent use.

Heretofore in the prior art, animated motion pictures have been utilized as visual aids in the teaching of geometry, trigonometry, etc. Such films are believed to have special value for promoting understanding by the student of various geometric concepts such as loci, processes, limits of geometric relationships, relationships existing between various geometric figures and shapes, etc. Such films are produced by making a large number of drawings and then filming selected ones of the drawings in a sequential manner substantially similar to the methods used to produce animated cartoons. Such a technique is relatively expensive due to the large number of drawings required and the length of time involved in filming a large number of such drawings sequentially.

Accordingly, an object of this invention is to overcome these and other disadvantages of the prior art.

Another object of this invention is to provide electrical means for providing an animated visual display of complex, dynamic, geometric figures and relationships.

Still another object of this invention is to provide a permanent record of dynamic geometric relationships, patterns, etc., caused to appear on a cathode ray tube oscilloscope.

A further object of this invention is to provide electrical apparatus for causing a repetitive sequence of geometric patterns to be generated on a display device in such a manner that only selected ones, or all, of said patterns can be made visible.

Still another object of this invention is to provide electrical apparatus for producing a plurality of repetitively occurring geometric patterns on a cathode ray oscilloscope in such a manner that the number of patterns and their sequence of occurrence may be readily varied.

A still further object of this invention is to provide electrical apparatus for producing a plurality of visual dynamic geometric patterns on a cathode ray oscilloscope in such a manner that only selected portions of each pattern capable of being produced are made visible.

Another object of this invention is to provide a repetitive sequence of a plurality of geometric patterns on a display device, such as a cathode ray oscilloscope, in such a manner as to produce a dynamic geometric figure wherein only selected patterns or portions thereof are visible thereby enabling dynamic geometric relationships to be visually presented in such a manner that they are readily and permanently recorded by suitable means such as motion picture film and video tape.

These and other objects of the present invention are accomplished with a visual display device, such as a cathode ray oscilloscope, having at least vertical and horizontal coordinate control means for producing a two dimensional visual display. First and second control means respectively are coupled to the vertical and horizontal coordinate control means. A plurality of electrical signals

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are coupled to the first and second control means which signals are each characterized as being capable of producing a predetermined pattern on the display device when applied, singularly or in combination with other of the signals, to the vertical and/or horizontal coordinate control means. Sequencing or cycling means are coupled to the vertical and horizontal control means. The sequencing means include a device, such as a ring counter, that produces a plurality of sequentially occurring signals or pulses in a repetitive manner. These sequentially occurring signals are selectively coupled to the vertical and horizontal control means to enable selected ones of said plurality of signals applied to said vertical and horizontal control means to be applied sequentially to said vertical and horizontal coordinate means of said display device to cause a plurality of sequentially occurring patterns to be capable of appearing on said display device. Intensity control means are also coupled to the display means for causing none, some, all, or only portions of the patterns capable of being displayed on said display device to be visible. The intensity control means also causes the display device to be blank during the time interval between successive patterns. The predetermined geometric figure thus produced on the display device is readily recorded by suitable means such as motion picture film and video tape.

This invention, as well as other advantages, features and objects thereof, will be readily apparent from consideration of the following detailed description relating to the annexed drawings in which:

FIGURE 1 illustrates, in block diagram form, one embodiment of the present invention;

FIGURE 2 illustrates a typical geometric relationship which may be readily generated in animated form by the present invention;

FIGURE 2A illustrates a modification of the geometric relationship shown in FIGURE 2;

FIGURE 3 illustrates, in block diagram form, another embodiment of the present invention capable of generating the geometric relationships illustrated in FIGURES 2 and 2A in an animated manner; and

FIGURE 4 illustrates the time relationship between various waveforms present at various points in the embodiment of the present invention illustrated in FIGURE 3.

Referring now to FIGURE 1, which illustrates in block diagram form one embodiment of the present invention, there is shown a visual display device, such as an oscilloscope 11, which includes a cathode ray tube having a screen or face 12 upon which geometric patterns or figures are caused to visually appear. In practicing the present invention, the cathode ray tube need only have one electron gun that produces a single electron beam that may impinge upon the screen or face 12 in a well known manner. The cathode ray tube contains vertical beam deflection means (not shown) coupled to the terminal 13 (Y input) and horizontal beam deflection means (not shown) coupled to the terminal 14 (X input) for controlling the point upon the face of the tube the electron beam impinges. As is well known to those skilled in the art, the face of the cathode ray tube may be considered as a two dimensional plane having vertical Y and horizontal X coordinates. The terminal 15 (Z input) of the oscilloscope is coupled to the cathode ray tube in such a manner that a potential applied to the terminal 15 controls the intensity of the light generated by the electron beam impinging upon the face 12. This can be accomplished, in a well known manner, by connecting the terminal 15 to the grid of the cathode ray tube.

In order to produce a predetermined geometric figure or pattern on the face 12 of the oscilloscope 11, it is necessary to apply the proper electrical signal or waveform to the vertical and/or horizontal deflection means

of the oscilloscope, such as by way of the terminals 13 and 14 respectively. For example, by applying a sine wave to the vertical input terminal 13 which is 90° out of phase with a sine wave applied to the horizontal terminal 14, a circle will appear on the face or screen. A vertical line appears on the face 12 of the cathode ray tube whenever a sawtooth wave is applied to the horizontal input terminal 14. Many other geometric shapes and figures are similarly generated on the face 12 of the oscilloscope. The waveforms required to generate such patterns and figures are well known or readily calculated by well known techniques.

In practicing the present invention, the waveforms or electrical signals required to create the desired individual geometric patterns to be produced on the screen 12 may be simultaneously and continuously generated by a function generator 16 shown on FIGURE 1. The various electrical signals are simultaneously applied to vertical and horizontal control means 17 and 18 by way of the leads 19 and 20 respectively. A time slot sequencer 21 is coupled to the vertical 17 and horizontal 18 control means by way of the leads 22 and 23 respectively.

The sequencer 21 functions to repetitiously produce a plurality of sequentially occurring control or timing pulses or signals that, when applied to the vertical control means 17 and horizontal control means 18 by way of the leads 22 and 23, cause the vertical and horizontal control means to sequentially apply selected ones of the signals appearing on the leads 19 and 20 to vertical input terminal 13 and horizontal input terminal 14, respectively, by way of the amplifiers 25 and 26. This causes a sequence of geometric patterns to visually appear on the face 12 of the cathode ray tube oscilloscope in a repetitive manner. By causing the total sequence of patterns to be rapidly repeated on the face 12, the composite geometric figure, composed of the plurality of sequentially-generated individual patterns, appears to the eye and other optical recording means as one entity.

The amplifiers 25 and 26 serve to amplify the signals they receive from the vertical 17 and horizontal 18 control means. By using the vertical and horizontal amplifiers contained within the oscilloscope 11, the amplifiers 25 and 26 can be eliminated. Also, if the signals sequentially leaving the vertical 17 and horizontal 18 control means are of sufficient amplitude, they may be applied directly to the vertical and horizontal deflection plates or coils of the cathode ray tube in a well known manner.

The time slot sequencer 21 also functions to apply a series of sequentially occurring blanking signals to the intensity control terminal 15 of the oscilloscope 11 by way of the lead 27. These blanking or intensity control signals serve to control the intensity of the electron beam generated by the cathode ray tube so that none, some, or all the individual patterns capable of being generated on the face 12 of the oscilloscope are caused to be visible. This can readily be accomplished by causing the intensity control terminal 15 to be coupled to the control grid of the cathode ray tube so that a negative potential appearing on the terminal 15 interrupts the electron beam generated by the electron gun of the cathode ray tube thereby causing the face 12 to be blank. Which, if any, of the patterns capable of being produced on the screen or face 12 are made visible can readily be controlled by manual switches (not shown) associated with the time slot sequencer 21. Alternatively, the manual switches can be replaced by more sophisticated equipment such as a computer.

By varying the amplitude of the signals produced by the function generator 16, the geometric figure illustrated on the screen or face 12 can be varied to illustrate various other relationships existing between the individual patterns that make up an entire geometric figure. As will be obvious to those skilled in the art in light of the above description, by being able to manipulate a composite fig-

ure at will, and by controlling the position and amplitude of the various individual patterns comprising the composite figure, the electrical apparatus of FIGURE 1, by the technique of superimposing simple patterns by means of time-sharing on a cathode ray tube oscilloscope, provides means for animating an unlimited number of algebraic, trigonometric and geometric relationships and patterns as well as providing means for exploring more complex mathematical processes involving complex variables and various mapping transformations and the like.

The time slot sequencer 21 shown in FIGURE 1 also produces pulses to blank the face 12 of the oscilloscope during the transition time between sequentially occurring patterns, to produce clearer composite figures on the face 12. If each individual pattern making up a composite complex geometric figure is generated on the face 12 in the same length of time, the cathode ray tube electron beam must move faster in order to generate certain patterns, such as a relatively large circle, than it does in generating other patterns, such as a short straight line segment. This can cause large, complex, individual patterns to have less light intensity than short less complex patterns. In order to correct this, a brightness control circuit 28 shown in FIGURE 1 may be used. The brightness control circuit 28 may operate so as to monitor the waveforms of the signals applied to the vertical 17 and horizontal 18 control means and produce a DC signal having a magnitude proportional to the rate of change of the individual patterns to be produced on the face 12 of the oscilloscope 11. This DC signal is applied to the intensity control terminal 15 of the oscilloscope 11 by way of the time slot sequencing circuit 21 and increases the intensity of the electron beam, and thus the brightness of the pattern on the face 12, whenever a complex pattern having a large sweep rate is being generated.

For example, a suitable brightness control device may include separate circuits for developing rate signals proportional to the rate of change of the trace along the X and the Y axis, circuits for separately squaring the rate signals, a circuit for adding the squared rate signals, and a circuit for developing a brightness signal proportional to the square root of the added squared rate signals.

The animated geometric figures caused to appear on the oscilloscope 11 are permanently recorded by recording means 29, such as a motion picture camera, having its lens portion 30 optically coupled to the face 12 of the oscilloscope. As long as the rate of generating a complete, repetitively occurring composite geometric figure is high enough, there is no necessity for synchronizing the individual patterns appearing on the face 12 with the motion picture camera. For example, if successive patterns are generated on the face 12 at a rate of one thousand times a second, a complete circle can be generated during each time period by a sine wave having 1052 radians per second. At sixteen frames per second a standard movie camera shutter is open approximately thirty-three milliseconds for each frame. During this period 5½ complete images are recorded. If, when generating very complex composite geometric patterns, etc., on the face 12, it is desired to synchronize each individual image appearing on the face with the camera 29, this can readily be accomplished due to the digital nature of the apparatus illustrated in FIGURE 1. Alternatively, the movie camera 29 can be replaced with a television camera which produces a permanent record on video tape.

The present invention will be more clearly understood by way of a specific example. Referring now to FIGURE 2, there is illustrated the bisection of a line segment 31 located between two points 32 and 33 which are the centers of two overlapping circles 34 and 35 respectively. A vertical line 36 drawn through the intersections 37 and 38 of the circles, bisects the line segment 31 located between the two points 32 and 33. The geometric relationship shown in FIGURE 2 is illustrated in modified

form in FIGURE 2A which shows only the portions of the two circles 32 and 33 near their points of intersection 37 and 38.

The geometric relationships illustrated in FIGURES 2 and 2A are readily animated in a dynamic manner by the apparatus of FIGURE 3. Reference to FIGURE 3 shows an oscilloscope 41 having a cathode ray tube screen or face 42 wherein the spot position on the face 42 is controlled in the X axis by means of a voltage on the horizontal input terminal 43, the spot position on the face 42 in the Y axis is controlled by means of a voltage on the vertical input terminal 45 and the brightness of the spot on the face 42 is controlled by a voltage on the intensity control or Z terminal 46.

The composite geometric figure shown in FIGURE 2 can be constructed on the oscilloscope 41 face of FIGURE 3 by sequentially generating, in rapid succession, six elementary figures. That is, the points 32 and 33, the two circles 34 and 35 centered on the points 32 and 33 respectively, the horizontal line 31 passing through the two points 32 and 33, and the vertical line 36 passing through the intersecting points 37 and 38 of the two circles. These figures may be sequentially generated on the oscilloscope 41 by sequentially applying the following voltage to the horizontal 43 and vertical 45 input terminals: The point 32 by  $X=-A$  and  $Y=0$  where  $A$  is a constant; the point 33 by  $X=+A$  and  $Y=0$ ; the circle 34 by  $X=-A+B \cos wt$  and  $Y=0+B \sin wt$  where  $B$  is a constant; the circle 35 by  $X=+A+B \cos wt$  and  $Y=0+B \sin wt$ ; the horizontal line 31 by a sawtooth wave  $X=C+Dt$  and  $Y=0$  where  $C$  is a constant and  $Dt$  increases as a function of time for a limited period of time; and the vertical line 36 by a sawtooth wave  $X=0$  and  $Y=E+ Ft$  where  $E$  is a constant and  $Ft$  increases as a function of time for a limited time period.

FIGURE 3 shows four analog gates 47, 48, 49 and 50 having their outputs commonly connected to the horizontal input terminal 43 of the oscilloscope 41 by way of the amplifier 51. Applied to the gates 47, 48, 49 and 50 are the  $-A$ ,  $B \cos wt$ ,  $+A$  and  $C+Dt$  voltages, respectively, which voltages occur simultaneously and continuously. Two analog gates 52 and 53 have their outputs commonly connected to the vertical input terminal 45 of the oscilloscope 41 by way of the amplifier 56. Applied to the gates 52 and 53 are the  $B \sin wt$  and  $E+ Ft$  voltages respectively. The gates 47, 48, 49 and 52 are opened, thereby enabling the voltages or signals applied thereto to be applied to the vertical 45 or horizontal 43 input terminals of the oscilloscope, by signals or timing pulses applied to the analog gates by way of the OR gates 57, 58, 59 and 60 respectively. The gates 50 and 53 are opened by a signal applied to them by way of the leads 61 and 62 respectively.

The signals or pulses which open the gates 47, 48, 49, 50, 52 and 53 are produced by sequencing means such as the ring counter 65 contained within the dashed lines. Such ring counters repetitively produce a plurality of sequentially occurring signals and are well known in the art. The ring counter has six stages 66, 67, 68, 69, 70 and 71, each corresponding to, or associated with, one or more of the six elementary patterns comprising the geometric figure shown in FIGURE 2. When generating other geometric figures, the ring counter can readily be adapted to short out unwanted stages so that the number of active stages corresponds to the number of separate patterns comprising a composite geometric figure. A ring counter, such as the one illustrated in FIGURE 3 operates such that one stage is in a state or condition which is different from the state of all the other stages. By applying an input pulse to each stage simultaneously, the single state or condition is caused to progress to the adjoining stage. By applying a plurality of sequentially occurring input pulses, the single state is caused to move around the ring counter.

The ring counter 65 shown in FIGURE 3 is driven by the clock generator 74 which may be any suitable device such as a free running multivibrator. The output of the clock generator is illustrated as a series of square waves 131 in FIGURE 4 which figure shows a plurality of waveforms at various points in the electrical apparatus of FIGURE 3 during the time required to generate one composite geometric figure (FIGURE 2) on the oscilloscope 41. Reference to FIGURES 3 and 4 shows that for each cycle produced by the clock generator 74, an output pulse is generated by one of the stages of the ring counter. For example, between times  $t_1$  and  $t_2$  an output pulse 132 is produced by the first stage 66, between time  $t_2$  and  $t_3$  an output pulse 133 is generated by the second stage 67, between time  $t_3$  and  $t_4$  an output pulse 134 is produced by the third stage 68, etc. The output pulses 132, 133, 134, 135, 136 and 137, associated with the stages 66, 67, 68, 69, 70 and 71, respectively, occur sequentially and once the sequence is completed it begins again as long as the clock pulses 131 produced by the clock generator 74 are applied to the ring counter.

The output pulses 132 through 137 produced by the ring counter 65 are applied to selected ones of the analog gates 47, 48, 49, 50, 52 and 53 by way of the leads 77 in such a manner that portions of the continuously occurring voltages or signals applied to the gates are caused to be sequentially applied to the oscilloscope 41 thereby generating the composite geometric figure shown in FIGURE 2 on the oscilloscope 41 face 42. For example, by tracing the leads 77 it will be seen that between time  $t_1$  and  $t_2$  the pulse 132 is applied, by way of the OR gate 57, to the gate 47 causing the  $-A$  voltage to create the point 32 of FIGURE 2 on the oscilloscope. At times  $t_2$  the pulse 132 terminates, terminating the point 32 on the oscilloscope, and between time  $t_2$  and  $t_3$  the pulse 133 is applied, by way of the OR gate 59, to the gate 49 which applies the voltage  $+A$  to the horizontal 43 input of the oscilloscope to create the point 33.

In like manner, between time  $t_3$  and  $t_4$  the pulse 134 is applied to the gates 48, 47 and 52 which applies the voltages  $B \cos wt$  and  $-A$  to the horizontal input terminal 43 and the voltage  $B \sin wt$  to the vertical input terminal 45 to create the circle 34 having its center at the point 32. In like manner, at time  $t_4$  to  $t_5$  the pulse 135 is applied to the gates 48, 52 and 49 which causes the voltages  $B \cos wt$ ,  $B \sin wt$  and  $+A$  to trace out the circle 35 having its center at point 33. Also, at time  $t_5$  to  $t_6$  the pulse 136 is applied to the gate 50 causing the voltage  $C+Dt$  to trace out the horizontal line 31. Finally, at time  $t_6$  to  $t_7$  the pulse 137 is applied to the gate 50 causing the voltage  $E+ Ft$  to trace out the vertical line 36. Once this sequence is completed, it is immediately repeated. By making the time required to complete each sequence high enough, the composite geometric figure illustrated in FIGURE 2, composed of the six elementary figures, is continuously visible on the oscilloscope 41 face 42.

In order to animate the composite geometric figure and to blank the face 42 of the oscilloscope during the time interval between successive patterns so as to produce a clear and distinct geometric figure, means for producing intensity control or blanking signals are provided which include a delay circuit 81, such as a monostable multivibrator, which produces a short pulse 138 for every square wave 131 produced by the clock generator 74. Reference to FIGURE 4 shows that each delay pulse 138 begins at time  $t_1$ ,  $t_2$ ,  $t_3$ ,  $t_4$ ,  $t_5$ , and  $t_6$ , and has a width which is a small portion of the total time required to generate a complete square wave 131 at the clock generator 74 of FIGURE 3. The delay circuit 81 is synchronized by the clock signals 131 which are applied to the delay circuit by way of the lead 82.

The delay pulses 138 of short time duration produced by the delay circuit 81 are applied to an AND gate 83 by way of the lead 84. Also, the output pulses 132

through 137 of each stage 66 through 71 of the ring counter 65 are applied to the AND gate 83 by way of OR gate 85 and a plurality of manual switches SW1, SW2, SW3, SW4, SW5, and SW6 respectively. The output of the AND gate 83 is applied to the set terminal of an intensity control or blanking circuit 86 by way of another OR gate 87. Circuit 86 may take the form of a bistable multivibrator type of circuit. The reset terminal of the intensity circuit is coupled to the pulses produced by the clock generator 74 by way of still another OR gate 88. The output of the intensity control circuit 86 is applied, by way of the lead 89, to the intensity control terminal 46 of the oscilloscope 41.

Assume now that each of the switches SW1 through SW6 are open and that the switch SW7 (the function of which will be described below) is also open. The switches SW1 through SW6 being open prevents any of the pulses 132 through 137, produced by the ring counter, to appear at the AND gate 83. Accordingly, each time a delay pulse 138 appears on the lead 84, it is not applied to the set terminal of the intensity control circuit 86 because no other signals are applied to the AND gate 83. Therefore, at the beginning, that is, at time  $t_1$ ,  $t_2$ ,  $t_3$ , etc., of each time period during which an individual pattern can be traced on the oscilloscope, the leading edge of the clock signals 131 appearing on the lead 90 sets the intensity control circuit into its reset position. Since no signals can be applied to the set terminal of the intensity control circuit to place it into its set state as long as the switches SW1 through SW7 are open, the intensity control circuit 86 remains in a reset state during which time it applies a negative voltage to the intensity control terminal 46 of the oscilloscope 41 which causes the face 42 of the oscilloscope to blank out. This is so even though the signals required to repetitively produce the sequence of the six patterns comprising the geometric figure shown in FIGURE 2 are being applied to the vertical 45 and horizontal 43 input terminals of the oscilloscope.

Assume now that the switch SW1 is closed. At time  $t_1$  the leading edge of a delay pulse 138 coincides with the leading edge of the control pulse 132 produced by the first stage 66 of the ring counter 65. This causes the AND gate 83 to pass a narrow pulse (not shown), having a width equal to a delay pulse 138, to the set terminal of the intensity control circuit 86. By differentiating this pulse and causing the voltage spike, produced by this differentiating, corresponding to the trailing edge of the pulse to trigger the intensity control circuit into its set state from its reset state, a positive voltage appears on the intensity control terminal 46 by way of the lead 89 which causes the trace on the face 42 of the oscilloscope to be visible. The reason for causing the trailing edge of the pulse appearing at the set terminal of the intensity control circuit to trigger the circuit 86 into its set state rather than the leading edge of the pulse will become apparent in conjunction with the description that follows.

At time  $t_2$  the leading edge of the delay pulse 138 appearing on the lead 90 will again place the intensity control circuit 86 into its reset state thereby blanking out the trace on the oscilloscope. As long as the remaining switches SW2 through SW7 are open, the trace remains blanked out until the next sequence at which time the trace again becomes visible at time  $t_1$  through  $t_2$ .

It is clear then, that by closing the switch SW1 the trace on the oscilloscope is made visible for substantially all of the time occurring between  $t_1$  and  $t_2$  except for a short time just subsequent to time  $t_1$ . As discussed hereinabove, between time  $t_1$  and  $t_2$  the point 32 of the geometric figure illustrated in FIGURE 2 is traced on the face of the oscilloscope.

Assume now that switches SW1 and SW2 are closed and that the remaining switches are open. During the time  $t_1$  to  $t_2$  the point 32 of FIGURE 2 is traced on the oscilloscope as discussed above. At time  $t_2$  the leading

edge of a clock signal 131 appearing on the lead 90 places the intensity control circuit into its reset state thereby producing a negative voltage on the lead 89 that blanks out the face of the oscilloscope. Also, at time  $t_2$ , however, a delay pulse 138 has its leading edge coinciding with the leading edge of the control signal 133, produced by the second stage 67 of the ring counter 65, at the input of the AND gate 83. This causes the AND gate to pass a short signal or pulse, having a time duration equal to a delay pulse, to the set terminal of the intensity control circuit by way of the OR gate 87.

As pointed out above, the trailing edge of this pulse, which occurs a short time after time  $t_2$ , and not its leading edge which occurs at time  $t_2$ , places the intensity control circuit back into its set state, thereby causing the trace of the oscilloscope to be visible.

At time  $t_3$  the leading edge of a clock pulse 131 again places the control circuit into its reset state and thereby blanks the trace of the oscilloscope. The trace will remain blanked out as long as the remaining switches SW3 through SW7 are closed.

During the next sequence, however, the trace will again be made visible for substantially all of the time between  $t_1$  and  $t_3$ . Therefore by closing the switches SW1 and SW2, the trace on the face of the oscilloscope is made visible for substantially all of the time occurring between time  $t_1$  and  $t_3$ .

As discussed hereinabove, the points 32 and 33 of the geometric figure shown in FIGURE 2 are traced during this time period. By causing the trailing edge of the pulse applied to the set terminal of the intensity control 86 to place the circuit 86 into the set state, the trace on the oscilloscope was blanked out for a length of time equal to the width of a delay pulse 131 beginning at time  $t_2$ . That is, the trace was blanked out during a short time interval between the successively occurring points 32 and 33 of FIGURE 2. Therefore, means have been provided to blank out the trace during a short time interval between successively occurring patterns to produce a clear and distinct composite geometric pattern.

Assume now that the switches SW1, SW2 and SW5 are closed and the remaining switches are open. At time  $t_1$  to  $t_3$  the points 32 and 33 of FIGURE 2 will be generated in a manner as discussed above. At time  $t_3$  the leading edge of a clock pulse on the lead 90 places the intensity control circuit 86 into its reset state thereby blanking out the trace of the oscilloscope. At time  $t_3$  a pulse of short duration also appears at the set terminal of the intensity control circuit 86 as described hereinabove. The trailing edge of this pulse places the intensity control circuit into its set state which causes the oscilloscope trace to be visible.

At time  $t_6$ , the leading edge of a clock pulse places the intensity control circuit into its reset state thereby blanking out the trace on the oscilloscope. During the next sequence the trace again will be visible during most of the time  $t_1$  to  $t_3$  and  $t_5$  to  $t_6$ . As discussed above, at times  $t_1$  to  $t_2$ ,  $t_2$  to  $t_3$  and  $t_5$  to  $t_6$  the points 32 and 33 and the horizontal line 31, respectively, are traced on the oscilloscope 41.

As is now obvious, any one or more of the six elementary patterns making up the composite geometric pattern shown in FIGURE 2 may be generated on the oscilloscope 41 by closing any one or more of the switches SW1 through SW6. Also, by manipulating the switches SW1 through SW6, the geometric figure shown in FIGURE 2, and any individual pattern which is a part thereof, can be arbitrarily manipulated at will. Also, the amplitude of the signals sequentially appearing on the vertical 45 and horizontal 43 input terminals of the oscilloscope 41 can be varied to illustrate various relationships between the individual patterns.

For example, by controlling the magnitude  $B$  of the two sine waves  $B \cos wt$  and  $B \sin wt$ , the two circles can be made to grow in size, overlap, then made to vary

in diameter together over a wide range. The voltage +A can be gradually varied to become -A, thereby bringing the two circles into coincidence, and the gradually brought back to +A. By gradually increasing B and making E more negative, the locus of the intersection of the two circles 34 and 35 is traced out until the vertical bisecting line 36 extends from the top to the bottom of the oscilloscope face. Accordingly, means have been provided for animating the geometric figure shown in FIGURE 2, in a dynamic manner, on an oscilloscope.

As discussed above, the geometric patterns appearing on the oscilloscope 41 may be permanently recorded for subsequent use by means of a television or motion picture camera. The waveforms illustrated in FIGURE 4 show the various waveforms in the apparatus of FIGURE 3 for one complete sequence when all of the switches SW1 through SW6 are closed.

Assume now that it is desirable that only the portions of the two circles 34 and 35 near the points 37 and 38 of their intersection be made visible in a manner as illustrated in FIGURE 2A. This can be accomplished by opening switches SW3 and SW4 and closing switch SW7. It will be assumed that switches SW1, SW2, SW5 and SW6 are closed; however, they need not be closed. At time  $t_3$  the leading edge of a clock pulse 131, shown in FIGURE 4, places the intensity control circuit 86 into its reset position thereby causing the trace on the oscilloscope to be blanked out. Since no control pulses 134 and 135 pass through the open switches SW3 and SW4 between times  $t_3$  to  $t_4$  and  $t_4$  to  $t_5$ , no pulse is applied to the set terminal of the intensity control circuit until time  $t_5$ .

Accordingly, the trace on the oscilloscope would appear to be blanked out entirely during the time  $t_3$  to  $t_5$  during which the two circles 34 and 35 are traced. This would be the case if it were not for the segmenter circuit 91 shown in FIGURE 3. During time  $t_3$  to  $t_4$  and  $t_4$  to  $t_5$ , the control pulses 134 and 135 generated by the third and fourth stages, respectively, of the ring counter are applied to the segmenter 91 to activate it.

The segmenter functions to place the intensity control circuit 86 into its set condition, thereby unblanking the trace, during the periods  $t_3$  and  $t_5$ , only when the amplitude of the signal appearing on the horizontal input terminal of the oscilloscope, which is applied to the segmenter by way of the lead 92, lies between  $\pm G$ , where G is a small arbitrary voltage. In other words, the segmenter 91 allows a selection of the signal to be displayed in accordance with the amplitude of the signal between the limits  $\pm 9$ , or for that matter between any two limits P and Q. If the signal has an amplitude between P and Q, then the total signal is displayed just as in the absence of the segmenter.

The output of the segmenter is applied to the OR gates 87 and 88 to control the voltage appearing on the lead 89 to blank the trace on the oscilloscope whenever the circle generating waveforms at the horizontal 43 and vertical 45 input terminals are more than G distant from the bisecting line 36 of FIGURE 2 at  $X=0$ . Therefore, means for generating the geometric figure shown in FIGURE 2A has been provided. That is, means are provided for causing only a portion of an individual pattern, making up a part of a composite geometric figure, to be made visible. The voltage levels appearing on the lead 89 when the switches SW3 and SW4 are open and the remaining switches, including switch SW7, are closed is illustrated in FIGURE 4 as the waveform 140. The segmenter can be constructed by appropriately interconnecting two comparators and two gates (not shown).

While the segmenter has been shown in connection with the X input, it is to be understood that a segmenter may be utilized in connection with the Y input in the same manner, and also that a number of X and Y seg-

menters can be utilized in combination, each one limiting the trace to applied signals between selected valves.

What has been described is electrical apparatus for producing animated, dynamic mathematical relationships and figures on visual display means, such as an oscilloscope, which figures and relationships can be permanently recorded on motion picture film or video tape. It is to be understood that the present invention is not limited to animating the geometric figures shown in FIGURES 2 and 2A, for many other figures and relationships may be animated in accordance with the teachings of the present invention. Also, the present invention is not limited to the specific interconnection of the specific logic circuits shown in FIGURE 3, for other circuits and interconnections can be used in practicing the present invention. For example, binary dividers and matrix summation of the outputs of the dividers can replace the ring counter 65 shown in FIGURE 3.

Having therefore disclosed hereinabove features and apparatus not heretofore known in the art, the appended claims are directed to those features of novelty believed to define the nature of the present invention.

What is claimed is:

1. Electrical apparatus for visually displaying and animating composite algebraic, trigonometric and geometric figures and relationships which are the superimposed composite of a plurality of unrelated and individual elemental patterns, said electrical apparatus comprising:
  - a visual display device including an oscilloscope having horizontal and vertical deflection input terminals and an intensity control terminal for respectively controlling the horizontal and vertical deflection and the intensity of the oscilloscope beam;
  - vertical control means including a plurality of analog gates having their outputs commonly coupled to said vertical input terminal;
  - horizontal control means including a plurality of analog gates having their outputs commonly coupled to said horizontal input terminal;
  - function generator means for generating a plurality of independent electrical signals, each of which, when applied to said deflection input terminals, produces one of said plurality of elemental patterns, said function generator means including means for independently varying at least one characteristic of at least one of said plurality of elemental patterns for producing animation, said electrical signals being applied to the analog gates of said vertical and horizontal control means in such a manner that each of said elemental patterns is individually displayable upon said display device upon selectively controlling said analog gates;
  - sequencing means for providing a plurality of sequentially occurring timing signals that are applied to selected ones of the gates of said vertical and horizontal control means, whereby each of the electrical signals characteristic of one of said elemental patterns is sequentially applied to the vertical and horizontal deflection input terminals of said display device and whereby the sequential application of each of said electric signals, one at a time, produces the superimposed composite of the unrelated and individual elemental patterns; and
  - intensity control means having an output coupled to said intensity control terminal, said intensity control means being responsive to said sequencing means and operative to blank the display when changing from one electrical signal to the next.
2. Electrical apparatus in accordance with claim 1 which further includes a switching means responsive to said sequentially occurring timing signals and an elemental pattern suppression signal and operative to provide an elemental pattern blanking signal for actuating said intensity control means, whereby the display of selected elemental patterns is suppressed.

3. Electrical apparatus in accordance with claim 1 which further includes segmenting means responsive to said sequentially occurring timing signals and selected ones of said electrical signals and operative to provide a segmentation blanking signal whenever the selected electrical signal being applied to said vertical and horizontal input terminals has an amplitude outside a selected range of amplitudes, said segmentation blanking signal being applied to the intensity control terminal for actuating the same.

4. Electrical apparatus in accordance with claim 2 in which said switching means is automatically reset by the next occurring timing signal.

5. Electrical apparatus in accordance with claim 1 in which said sequencing means comprises a ring counter and a source of clock pulses for driving said ring counter.

6. An electrical apparatus in accordance with claim 5 wherein said intensity control means is set by said clock pulses, said intensity control means further including a delay means for providing a reset pulse for resetting said intensity control means.

7. An apparatus in accordance with claim 1 wherein a recording means is optically coupled to said visual display device for producing a permanent record of the display.

8. An electrical apparatus in accordance with claim 2 in which said switch means includes a plurality of manually settable switches for developing said elemental pattern suppression signals.

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